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ORIGINAL ARTICLE

From Numeric Models to Granular System Modeling

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Abstract In the era of advanced methodologies and practices of system modeling, we are faced with ever growing challenges of building models of complex systems that are in full rapport with reality. These challenges are multifaceted. Human centricity becomes of paramount relevance in system modeling and because of this models need to be customized and easily interpretable. More and more visibly, experimental data and knowledge of varying quality being directly acquired from experts have to be efficiently utilized in the construction of models. The quality of data and ensuing quality of models have to be prudently quantified. There are ongoing and even exacerbated challenges to build intelligent systems, modeling multifaceted phenomena, and deliver efficient models that help users describe and understand systems and support processes of decision-making. We have to become fully cognizant that processing and modeling has to be realized with the use of entities endowed with well-defined semantics, namely information granules. Human do not perceive reality and reason in terms of numbers but rather utilize more abstract constructs (information granules), which are helpful in setting up a certain cognitive perspective and ignore irrelevant details when dealing with the complexity of the systems.

To make this study self-contained, we briefly recall the key concepts of granular computing and demonstrate how this conceptual framework and its algorithmic fundamentals give rise to granular models. We discuss several representative formal setups used in describing and processing information granules including fuzzy sets,

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rough sets, and interval calculus. Key architectures of models dwell upon relationships among information granules. We demonstrate how information granularity and its optimization can be regarded as an important design asset to be exploited in system modeling and giving rise to granular models. With this regard, an important category of rule-based models along with their granular enrichments is studied in detail.

Keywords Granular computing · Granular models · Information granularity · Allocation of information granularity · Rule-based model

1. Introduction

In system modeling, it is evident that there are no ideal models. Numeric data are not ideally (without any error) captured by any model, no matter how complex the model is. As usual, by adhering to the Ockahms razor principle, we strive to build simple models and establish a crucial balance between accuracy and simplicity requirements. In spite of the diversity of the architectures of the models, especially those emerging in the realm of computational intelligence the apparent challenges remain. An interesting, innovative, and promising direction is to pursue conceptualizing and building models that are formed at the higher level of abstraction and in this way become capable of coping with the essentials of the system to be modeled. These models are constructed in terms of information granules and in the sequel are referred to as granular models. They can also realize a generalization of existing numeric models. Information granules are formalized in various settings as sets (intervals), fuzzy sets, rough sets, etc. Depending upon the nature of the model, we can talk about granular neural networks, granular regression models, etc.

The objective of this study is to conceptualize an idea of treating information granularity as an essential design asset in system modeling, which, when used properly can make the model to be in line with the complexity of the problem (system) and gives rise to a hierarchy of granular models depending upon a level of specificity of information granularity.

The study is organized in the following way. To make the paper self-contained, we start with a brief prerequisite material on granular computing (Section 2). In Sections 3 and 4, we look into a characterization of information granules by discussing their two important characterizations such as coverage abilities (related with the generalization facet of information granules) and specificity description. The concept of an optimal allocation of information granularity sought as a crucial design asset in system modeling is introduced in Section 5. The discussion covered in Section 6 brings this notion directly to system modeling while a number of protocols of allocation of information granularity are outlined in Section 7. A detailed discussion on granular rule-based models is covered in Section 8. Conclusions are offered in Section 9.

2. Granular Computing: An Introduction

To make the study presented here self-contained and offer a better focus of the paper, we present a concise introduction to granular computing [4, 11, 17-19] being regarded as a formal vehicle to cast data analysis and modeling tasks in a certain conceptual framework.

Information granules are intuitively appealing constructs, which play a pivotal role in human cognitive and decision-making activities. We perceive complex phenomena by organizing existing knowledge along with available experimental evidence and structuring them in a form of some meaningful, semantically sound entities, which are central to all ensuing processes of describing the world, reasoning about the environment and support decision-making activities. The term information granularity itself has emerged in different contexts and numerous areas of application. It carries various meanings. One can refer to artificial intelligence in which case information granularity is central to a way of problem solving through problem decomposition where various subtasks could be formed and solved individually. In general, by information granule one regards a collection of elements drawn together by their closeness (resemblance, proximity, functionality, etc.) articulated in terms of some useful spatial, temporal, or functional relationships. Subsequently, granular computing is about representing, constructing, and processing information granules.

We can refer here to some areas, which offer compelling evidence as to the nature of underlying processing and interpretation in which information granules play a pivotal role: image processing, processing and interpretation of time series, granulation of time, design of software systems. Information granules are examples of abstractions. As such they naturally give rise to hierarchical structures: The same problem or system can be perceived at different levels of specificity (detail) depending on the complexity of the problem, available computing resources, and particular needs to be addressed. A hierarchy of information granules is inherently visible in processing of information granules. The level of detail (which is represented in terms of the size of information granules) becomes an essential facet facilitating a way a hierarchical processing of information with different levels of hierarchy indexed by the size of information granules.

Even such commonly encountered and simple examples presented above are convincing enough to lead us to ascertain that (a) information granules are the key components of knowledge representation and processing, (b) the level of granularity of information granules (their size, to be more descriptive) becomes crucial to the problem description and an overall strategy of problem solving, (c) hierarchy of information granules supports an important aspect of perception of phenomena and deliver a tangible way of dealing with complexity by focusing on the most essential facets of the problem, (d) there is no universal level of granularity of information; essentially the size of granules becomes problem-oriented and user dependent.

There are several well-known formal settings in which information granules can be expressed and processed:

Sets (intervals) realize a concept of abstraction by introducing a notion of dichotomy: We admit element to belong to a given information granule or to be excluded from it. Along with set theory comes a well-developed discipline of interval analysis. Alternatively to an enumeration of elements belonging to a given set, sets are described by characteristic functions taking on values in $\{0, 1\}$.

Fuzzy sets [13, 18] provide an important conceptual and algorithmic generalization of sets. By admitting partial membership of an element to a given information granule we bring an important feature which makes the concept to be in rapport with reality. It helps working with the notions where the principle of dichotomy is neither justified nor advantageous. The description of fuzzy sets is realized in terms of membership functions taking on values in the unit interval. Formally, a fuzzy set A is described by a membership function mapping the elements of a universe X to the unit interval [0, 1].

Shadowed sets [12] offer an interesting description of information granules by distinguishing among elements, which fully belong to the concept, are excluded from it and whose belongingness is completely unknown. Formally, these information granules are described as a mapping X: $\rightarrow X$ $\{1, 0, [0,1]\}$ where the elements with the membership quantified as the entire [0, 1] interval are used to describe a shadow of the construct. Given the nature of the mapping here, shadowed sets can be sought as a granular description of fuzzy sets where the shadow is used to localize unknown membership values, which in fuzzy sets are distributed over the entire universe of discourse. Note that the shadow produces non-numeric descriptors of membership grades.

Probability-oriented information granules are expressed in the form of some probability density functions or probability functions. They capture a collection of elements resulting from some experiment. In virtue of the concept of probability, the granularity of information becomes a manifestation of occurrence of some elements. For instance, each element of a set comes with a probability density function truncated to [0, 1], which quantifies a degree of membership to the information granule.

Rough sets [7, 8, 11] emphasize a roughness of description of a given concept X when being realized in terms of the indiscernibility relation provided in advance. The roughness of the description of X is manifested in terms of its lower and upper approximations of a certain rough set. One can refer to a plethora of applications.

3. Information Granules: Coverage and Specificity

From the perspective of this study, there are two important and directly applicable characterizations of information granules, namely coverage and specificity [11].

Coverage, the concept of coverage of information granule, $\mathrm{cov}(\cdot)$ is discussed with regard to some experimental data existing in R^n, that is $\{x_1, x_2, \cdots, x_N\}$ and as the name stipulates, is concerned with its ability to represent (cover) these data. In general, the larger number of data is being "covered", the higher the coverage measure. Formally, the coverage is a non-decreasing function of the number of data that are represented by the given information granule A. Depending upon the nature of information granule, the definition of $\mathrm{cov}(A)$ can be properly formulated. For instance, when dealing with a multidimensional interval (hypercube) A, $\mathrm{cov}(A)$ in its normalized form is related with the cardinality of the data belonging to A, $\mathrm{cov}(A) = \frac{1}{N}\mathrm{card}\{x_k \mid x_k \in A\}$. For fuzzy sets, the coverage is realized as a σ-count of A, where we summed up the degrees of membership of x_k to A, $\mathrm{cov}(A)=\frac{1}{N}\sum_{k=1}^{N} A(x_k)$.

Specificity Intuitively, the specificity relates to a level of abstraction conveyed by the information granules. The higher the specificity, the lower the level of abstraction.

The monotonicity property holds: If for the two information granules A and B one has $A \subset B$ (when the inclusion relationship is articulated according to the formalism in which A and B are expressed), then specificity, sp($\cdot$) satisfies the following intuitively appealing inequality: sp(A) $\geq$ sp(B). Furthermore for a degenerated information granule comprising a single element x_0 we have a boundary condition sp($\{x_0\}$)=1. In case of a one-dimensional interval information granules, one can contemplate expressing specificity on a basis of the length of the interval, say sp(A) = exp(-length(A)); obviously, the boundary condition specified above holds here. If the range of the data is available (it could be easily determined), say, then sp(A) = $1 - |b - a|/range$ where A=$[a, b]$, $range$ = [$\min_k x_k$, $\max_k x_k$].

The realizations of the definitions can be augmented by some parameters that contributes to their flexibility. It is also intuitively apparent that these two characteristics are associated: The increase in one of then implies a decrease in another: An information granule that "covers" a lot of data cannot be overly specific and vice versa.

4. Information Granules of Higher Type

By information granules of higher type (2$^{\text{nd}}$ type and n^{th} type, in general) we mean granules in the description of whose we use information granules rather than numeric entities. For instance, in case of type-2 fuzzy sets we are concerned with information granules-fuzzy sets whose membership functions are granular. As a result, we can talk about interval-valued fuzzy sets, fuzzy sets (or fuzzy^2 sets, for brief), probabilistic sets and alike. The grade of belongingness are then intervals in [0, 1], fuzzy sets with support in [0, 1], probability functions truncated to [0, 1], etc. In case of type-2 intervals we have intervals whose bounds are not numbers but information granules and as such can be expressed in the form of intervals themselves, fuzzy sets, rough sets or probability density functions. Information granules of higher order are those whose description is realized over a universe of discourse whose elements are information granules. In some sense, rough sets could be sought as information granules of order-2. Information granules have been encountered in numerous studies reported in the literature; in particular stemming from the area of fuzzy clustering [5, 10] in which fuzzy clusters of type-2 have been investigated [6] or they are used to better characterize a structure in the data and could be based upon the existing clusters [14].

5. Optimal Allocation of Information Granularity

Information granularity is an important design asset in system modeling. Information granularity allocated to the original numeric construct elevates a level of abstraction (generalizes) of the original construct developed at the numeric level. It helps the original numeric constructs cope with the nature of the data. A way in which such an asset is going to be distributed throughout the construct or a collection of constructs to make the abstraction more efficient, is a subject to optimization. We start with a general formulation of the problem and then show selected realizations of the granular mappings.

Let us consider a certain multivariable mapping $y = f(x, a)$ with a being an n-dimensional vector of parameters of the mapping. The mapping can be sought (realized) as a general construct. One may think of a fuzzy model, neural network, polynomial, differential equation, linear regression, etc. The granulation mechanism G is applied to a. It gives rise to its granular counterpart, $A = G(a)$. Subsequently, this mapping can be described formally as follows

$$Y = G(f(x,a)) = f(x,G(a)) = f(x,A). \tag{1}$$

Given the diversity of the underlying constructs as well as a variety of ways information granules can be formalized, we arrive at a suite of interesting constructs including granular neural networks, say interval neural networks, fuzzy neural networks, probabilistic neural networks. Likewise we can talk about granular (fuzzy, rough, probabilistic $\cdots$) regression, cognitive maps, fuzzy models, just to name several constructs.

There are a number of well-justified and convincing arguments behind elevating the level of abstraction of the existing constructs. Those include: An ability to realize various mechanisms of collaboration, quantification of variability of sources of knowledge considered, better modelling rapport with systems when dealing with non-stationary environments.

Information granularity supplied to form a granular construct is a design asset whose allocation throughout the mapping can be guided by certain optimization criteria. Let us discuss the underlying optimization problem in more detail. In addition to the mapping itself, we are provided with some experimental evidence in the form of input-output pairs $D = (x_k, t_k), k = 1, 2, \cdots, N$. Given is a level of information granularity ε, $\varepsilon \in [0, 1]$. We allocate the available level ε to the parameters of the mapping so that the some optimization criteria are satisfied while the allocation of granularity satisfies the following balance $n\varepsilon = \sum_{i=1}^{n} \varepsilon_i$ where ε_i is a level of information granularity associated with the i-th parameter of the mapping. For further processing all the individual allocations are organized in a vector format $(\varepsilon_1\ \varepsilon_2\ \cdots \varepsilon_n)^T$.

The first criterion is concerned with the coverage of the output data t_k by the outputs produced by the granular mapping. For x_k we compute $Y_k = f(x_k, G(a))$ and determine a degree of inclusion of t_k in information granule Y_k, namely $\mathrm{incl}(t_k, Y_k) = t_k \subset Y_k$. Then we compute an average sum of the degrees of inclusion taken over all data, that is $Q = \frac{1}{N} \sum_{k=1}^{N} \mathrm{incl}(t_k, Y_k)$. Depending upon the formalism of information granulation, the inclusion returns a Boolean value in case of intervals (sets) or a certain degree of inclusion in case of fuzzy sets. Alluding just to sets and fuzzy sets as the formal models of information granules, the corresponding expressions of the performance index are expressed as follows,

- for sets (intervals)

$$Q = \frac{\mathrm{card}\{t_k \mid t_k \in Y_k\}}{N}, \tag{2}$$

- for fuzzy sets

$$Q = \frac{\sum_{k=1}^{N} Y_k(t_k)}{N}. \tag{3}$$

Here $Y_k(t_k)$ is a degree of membership of t_k in the fuzzy set of the information granule Y_k.

The second criterion of interest is focused on the specificity of Y_k – we want it to be as high as possible. The specificity could be viewed as a decreasing function of the length of the interval in case of set-based information granulation. For instance, one can consider the inverse of the length of Y_k, for instance $1/\text{length}(Y_k)$, $\exp(-\text{length}(Y_k))$, etc. In case of fuzzy sets, one can consider the specificity involving the membership grades. The length of the fuzzy set Y_k is computed by integrating the lengths of the β-cuts, $\int_0^1 \text{length}(Y_k^{\beta})\beta d\beta$.

More formally, the two-objective optimization problem is formulated as follows. Distribute (allocate) a given level of information granularity ε so that the following two criteria are maximized

$$\begin{aligned} &\text{Max} \quad \frac{1}{N}\sum_{k=1}^{N} \text{incl}(t_k, Y_k) \\ &\text{Max} \quad g(\text{length}(Y_k)) \ \text{(where } g \text{ is a decreasing function of its argument)} \\ &\text{s.t.} \quad n\varepsilon = \sum_{i=1}^{n} \varepsilon_i. \end{aligned} \tag{4}$$

A simpler optimization scenario involves a single coverage criterion. It can be regarded as an essential criterion considered in the problem

$$\begin{aligned} &\text{Max} \quad \frac{1}{N}\sum_{k=1}^{N} \text{incl}(t_k, Y_k) \\ &\text{s.t.} \quad n\varepsilon = \sum_{i=1}^{h} \varepsilon_i. \end{aligned} \tag{5}$$

There is an interesting monotonicity property: Higher values of lead to higher values ε of the maximized objective function, refer to Fig. 1. There could be different patterns of changes of Q versus ε as illustrated in the same figure. Typically, some clearly visible "knee" points are encountered on the curve beyond which the changes (increases) of Q become quite limited.

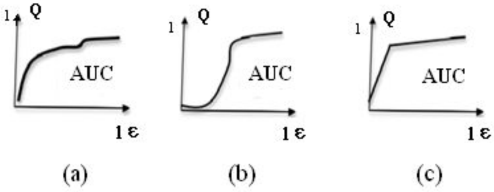

Fig. 1 Values of the coverage criterion Q regarded as a function of the assumed level of granularity ε. Shown are different relationships $Q(\varepsilon)$ with some "knee" points; (a), (b), (c)

By taking into account the nature of the relationship shown in these figures, we can arrive at some global view at the relationship that is independent from a specific value of ε. This is accomplished by taking an area under curve (AUC) computed as AUC = $\int_0^1 Q(\varepsilon)d\varepsilon$. The higher the value of the AUC, the better the performance of the granular mapping. Alternatively, one can consider a plot of the two characteristics of the mapping in the coverage-specificity plane and quantify the performance of the mapping using the AUC of these curves, Fig. 2.

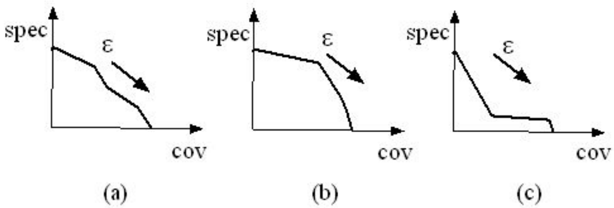

Fig. 2 Selected characteristics of coverage-specificity of granular models: (a) monotonic behavior of the relationship with the changes of ε, (b) increase of coverage and retention of specificity with the increase of ε, (c) rapid drop in specificity for increasing values of ε

6. An Emergence of Granular Models: Structural Developments

The concept of the granular models form a generalization of numeric models no matter what their architecture and a way of their construction are. In this sense, the conceptualization offered here are of general nature. They also hold for any formalism of information granules. A numeric model M_0 constructed on a basis of a collection of training data $(x_k, target_k)$, $x_k \in R^n$ and $target_k \in R$ comes with a collection of its parameters a_{opt} where $a \in R^p$. Quite commonly, the estimation of the parameters is realized by minimizing a certain performance index Q (say, a sum of squared error between $target_k$ and $M_0(x_k)$), namely $a_{opt} = \arg \mathrm{Min}_a Q(a)$. To compensate for inevitable errors of the model (as the values of the index Q are never equal identically to zero), we make the parameters of the model information granules [9], resulting in a vector of information granules $A = (A_1 \ A_2 \ \cdots \ A_p)$ built around original numeric values of the parameters a. The elements of the vector a are generalized, the model becomes granular and subsequently the results produced by them are information granules. Formally speaking, we have

- granulation of parameters of the model $A = G(a)$ where G stands for the mechanisms of forming information granules, viz. building an information granule around the numeric parameter.

- result of the granular model for any x producing the corresponding information granule Y, $Y = M_1(x, A) = G(M_0(x)) = M_0(x, G(a))$.

Information granulation is regarded as an essential design asset. By making the results of the model granular (and more abstract in this manner), we realize a better alignment of $G(M_0)$ with the data. Intuitively, we envision that the output of the granular model "covers" the corresponding target. Formally, let $\mathrm{cov}(target, Y)$ denote a certain coverage predicate (either Boolean or multivalued) quantifying an extent to which target is included (covered) in Y. The optimization is realized as discussed in the previous section.

7. Protocols of Optimal Allocation of Information Granularity

The numeric parameters of the model are to be made granular having at our disposal a level of information granularity viewed as the essential design asset. In what follows, to illustrate the idea, we consider interval information granules spanned over the numeric values. The allocation of information granularity can be realized in many different ways by engaging various levels of sophistication. The series of protocols presented below is organized with the increasing level of flexibility each of them supporting a better usage of information granularity:

P_1: uniform allocation of information granularity. This protocol is the simplest one. It does not invoke any optimization mechanism. All numeric values of the parameters are treated in the same way and become replaced by intervals of the same length. Furthermore the intervals are distributed symmetrically around the original values of the parameters.

P_2: uniform allocation of information granularity with asymmetric position of intervals around the numeric parameter. Here we encounter some level of flexibility: even though the intervals are of the same length, their asymmetric localization brings a certain level of flexibility, which could be taken advantage of during the optimization process. More specifically, we allocate the intervals of lengths $\varepsilon\gamma$ and $\varepsilon(1-\gamma)$ to the left and to the right from the numeric parameter where $\gamma \in [0, 1]$ controls asymmetry of localization of the interval whose overall length is ε. Another variant of the method increases an available level of flexibility by allowing for different asymmetric localizations of the intervals that can vary from one parameter to another. Instead of a single parameter of asymmetry (γ), we admit individual γ_i for various numeric parameters.

P_3: non-uniform allocation of information granularity with symmetrically distributed intervals of information granules. Each parameter of the model is endowed with the individual level of information granularity ε_i.

P_4: non-uniform allocation of information granularity with asymmetrically distributed intervals of information granules. Among all the protocols discussed so far, this one exhibits the highest level of flexibility.

P_5: An interesting point of reference, which is helpful in assessing a relative performance of the above methods, is to consider a random allocation of granularity. By doing this, one can quantify how the optimized and carefully thought out process of granularity allocation is superior over a purely random allocation process.

While the allocation of information granularity realized above through a collection of protocols offers several main strategies, some of the implementation details are dependent on the nature of the model. For instance if all parameters of the model are in the same range, say [0, 1] (as encountered in fuzzy neural networks operating logic operators), then the intervals around the numeric parameter ai are formed as shown above, namely

P_1 $[a_i - \varepsilon/2, a_i + \varepsilon/2]$,
P_2 $[a_i - \varepsilon\gamma, a_i + \varepsilon(1-\gamma)]$ or $[a_i - \varepsilon\gamma\ \ , a_i + \varepsilon(1-\gamma\ \ )]$,
P_3 $[a_i - \varepsilon\ \ /2, a_i + \varepsilon\ \ /2]$.

In case the parameters of the model are localized in different ranges, the realization of the intervals involves the magnitude of the parameters, say for P_1

$$[a_i(1-\varepsilon/2), a_i(1+\varepsilon/2)] \text{ if } a_i \neq 0 \text{ and } [a_i - \varepsilon/2, a_i + \varepsilon/2] \text{ if } a_i = 0.$$

8. Granular Rule-based Models

These functional rules (Takagi-Sugeno format of the conditional statements) link any input space with the corresponding local model whose relevance is confided to the region of the input space determined by the fuzzy set standing in the input space (A_i). The local character of the conclusion makes an overall development of the fuzzy model well justified: we fully adhere to the modular modeling of complex relationships. The local models (conclusions) could vary in their diversity; in particular local models in the form of linear functions (f_i) are usually of interest

$$\text{if } x \text{ is } A_i \text{ then } y \text{ is } f_i. \tag{6}$$

These models are then equivalent to those produced by the Mamdani-like rules with a weighted scheme of decoding (defuzzification). There has been a plethora of design approaches to the construction of rule-based models, cf. [1-3]

Information granularity emerges in fuzzy models in several ways by being present in the condition parts of the rules, their conclusion parts and both. In a concise way, we can describe this in the following way (below the symbol G(·) underlines the granular expansion of the fuzzy set construct abstracted from their detailed numeric realization or a granular expansion of the numeric mapping).

(i) *Information granularity associated with the conditions of the rules.* We consider the rules coming in the format

$$\text{if } G(A_i) \text{ then } f_i, \tag{7}$$

where $G(A_i)$ is the information granule forming the condition part of the i-th rule. An example of the rule coming in this format is the one where the condition is described in terms of a certain interval-valued fuzzy set or type-2 fuzzy set, $G(A_i)$.

(ii) *Information granularity associated with the conclusion part of the rules.* Here the rules take on the following form

$$\text{if } x \text{ is } A_i \text{ then } G(f_i) \tag{8}$$

with $G(f_i)$ being the granular local function. The numeric (linear) mapping f_i is made more abstract by admitting granular parameters. For instance instead of f_i we consider $G(f_i)$ where $G(f_i)$ is an interval or a linear function whose parameters are fuzzy numbers.

(iii) *Information granularity associated with the condition and conclusion parts of the rules.* This forms a general version of the granular model and subsumes the two situations listed above. The rules read now as follows

$$\text{if } G(A_i) \text{ then } G(f_i). \tag{9}$$

The augmented expression for the computations of the output of the model generalizes the expression used in the description of the fuzzy models (3). We have

$$Y = \sum_{\substack{i=1 \\ \oplus}}^{c} (GA_i(x) \otimes G(f_i)), \tag{10}$$

where the algebraic operations shown in circles and reflect that the arguments are information granules instead of numbers (say, fuzzy numbers). The detailed calculations depend upon the formalism of information granules being considered. Let us stress that Y is an information granule. Obviously, the aggregation given by (6) applies to (i) and (ii) as well; here we have some simplifications of the above stated formula. The two commonly used formalisms already reported in the literature are interval-valued fuzzy sets and type 2 fuzzy sets [11].

The design of the fuzzy rule-based model (3) splits into the two main conceptual and algorithmic phases, namely (a) construction of the condition parts, and (b) design of conclusions of the rules. As far as the formation of the condition part goes, the commonly accepted approach is to engage some methods of fuzzy clustering, say fuzzy c-means (FCM) [5, 10]. It is instructive to recall some main findings with this regard. Given experimental data, one forms a collection of clusters $A_1, A_2, \cdots, A_c$ by minimizing a certain objective function. The number of rules is equal to the number of clusters. The membership functions of $A_1, A_2, \cdots, A_c$ are a result of the optimization and are described by the well-known formula encountered in the FCM algorithm

$$A_i(x) = \frac{1}{\sum_{j=1}^{c} \left(\frac{\|x - v_i\|}{\|x - v_j\|} \right)^{2/(p-1)}}, \tag{11}$$

where p is a fuzzification coefficient, $p > 1$. The distance between the input x and the i-th prototype v_i is denoted by $\|\cdot\|$. The above expression determines the membership degree on a basis of distances between the prototype and the input of interest. The common choice in the FCM is the use of the Euclidean distance or its generalized (say, weighted) version.

Proceeding with the above stated scenarios of forming granular rule-based models, we can offer some detailed pointed:

(i) To incorporate information granularity into the condition part of the rules means that the results produced by fuzzy clusters are made granular. In other words, a method of granular fuzzy clustering can be involved here. For instance, FCM can be run considering a range of the values of the fuzzification coefficient [6] leading to the interval-valued membership grades of A_i. Some other option was presented in [14] where interval-valued prototypes were formed around the numeric values of the original prototypes subsequently producing intervals of membership values of A_{iS}.

(ii) The conclusion part is made granular by admitting granular parameters of the local functions. The optimization there involves.

(iii) This option involves both (i)-(ii) when forming information granules.

9. Conclusion

In granular computing, we strive to build a coherent and algorithmically sound processing platform. The mechanism of optimal allocation of information granularity provide a way of forming information granules and exploiting information granularity as an important design asset in a variety of models. In this context, we highlight an important role of information granules as a vehicle through which we can achieve higher performance of the models.

The idea of optimal allocation (distribution) of information granularity, which is one of the underlying principles of Granular Computing, calls for more advanced techniques of optimization (that go far beyond gradient-based techniques). In particular, one can anticipate the usage of evolutionary optimization techniques or swarm optimization methods [15, 16]. In this sense, we start witnessing here yet another example of an important synergy of technologies of computational intelligence. The granular constructs open a new avenue of granular computational intelligence in which information granularity starts playing a visible role in the design of collaborative intelligent systems.

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